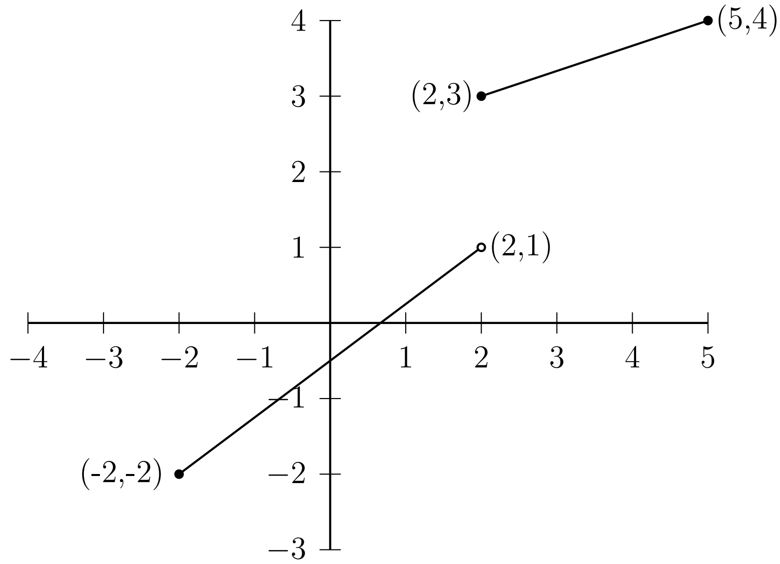
Exercise IV

1. Sketch the graph of a function $f(x)$ with domain [3,6] and range [2,7].

2. Sketch the graph of a function $f(x)$ with domain [3,6] and range $[2,7] \setminus \{4\}$.

3. The following is a sketch of the graph of the function $f(x)$:



   As the sketch indicates, f has domain [-2,5] and the graph consists of two straight line segments. Write down the definition for $f(x)$.

4. Given that $\lim\limits_{n\to\infty} \frac{1}{n} = 0$ and $\lim\limits_{n\to\infty} k = k$ for every constant sequence $\{k\}$, use the properties of limits to prove carefully that

$$\lim_{n\to\infty} \frac{3n^3 - 3n^2 + 1}{n^3 + n} = 3.$$

5. Let $k > 0$. Use the inequality $(1+k)^n \geq 1 + nk$, $\forall n \in \mathbb{N}$ and the *Squeezing Theorem* to prove that
   (i) $\lim\limits_{n\to\infty} \frac{1}{10^n} = 0$ (ii) $\lim\limits_{n\to\infty} 0.6^n = 0$ (iii) $\lim\limits_{n\to\infty} \left(\frac{4}{7}\right)^n = 0$

6. Determine whether each of the following sequences is convergent or not and, if convergent, determine the limit:
   (i) $\left\{\frac{3n}{5n^2+4}\right\}$, (ii) $\left\{\frac{6n^2+1}{n^3+5} + 7\right\}$, (iii) $\left\{\left(\frac{7}{9}\right)^n\right\}$, (iv) $\left\{\frac{3^n}{4^n}\right\}$,
   (v) $\left\{2 + \frac{6}{5^n}\right\}$, (vi) $\left\{\frac{4^n}{3^n}\right\}$, (vii) $\left\{\frac{n^2}{n+5}\right\}$.

7. Express each of the following infinite decimals in the form $\frac{a}{b}$ where a, b are integers:
   (i) $0.23\dot{1}$, (ii) $0.459\dot{3}\dot{5}\dot{1}$ (iii) $1.83\dot{9}$.

8. Consider the series

$$\frac{9}{10^2} + \frac{9}{10^4} + \frac{9}{10^6} + \cdots.$$

   Write down the first five terms of the sequence of partial sums for this series. Express each of these terms as a decimal.